Financial Constraints and Firm Dynamics

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May 10, 2010

VERY PRELIMINARY DRAFT - DO NOT QUOTE

Abstract

Empirical investigations show that the ability of firms to raise external financial resources represents a key factor influencing several dimensions of their dynamics. Motivated by these findings, we present a simple framework which allows for a unified analysis of the effects of financing constraints (FC) on the empirical distributions of firm size and growth rate, over different time horizons. As a measure of FC we take an official credit rating index, which directly measures the cost and scope of corporate access to capital markets.

Our findings suggest that FC play a relevant role both in the short and in long run. Over the short run, they operate through asymmetric "threshold effects", either preventing financially constrained firms from exploiting attractive growth opportunities, or further deteriorating the growth prospects of already slow growing firms. These effects get translated into different patterns of growth and result, over the longer run, into size distributions which evolve differently for firms affected by different degrees of financial constraints. In particular, we show that, in the presence of FC, the relation between firm size and the variability of its growth rates is moderated, while the natural auto-regressive structure of growth process get enhanced.

The main conclusions of the paper survive the inclusion in the model of other relevant determinants of the size-growth dynamics, like firm age, and the availability of internal financial resources.

1 Introduction

Firms' ability to raise external financial resources represents a factor influencing several dimensions of firm dynamics, as the links between financial and operational activities of firms are multiple and complex. These involves decisions pertaining to investment strategies, ability to enter and survive in a market, job creation and destruction, and internationalization. Within this broad research area, there exists a relatively well developed tradition of empirical studies which have sought to identify the effect of financing problems on size-growth trajectories of firms (for reviews, see Whited, 2006, Fagiolo and Luzzi, 2006, Oliveira and Fortunato, 2006). This paper contributes to this literature addressing two major difficulties, inherently related with the analysis of financial constraints (FC) effects, which previous studies have either solved in inconclusive ways, or completely overlooked.

A first problem concerns the intrinsically difficult task of measuring FC, not only within the context of size-growth dynamics, but more generally. This stems from the simple consideration that financial constraints are not directly observable, as one does not know whether banks or other financial institutions have refused a loan or if abnormally high interest rate are imposed to a particular firm. To overcome this problem researchers have typically followed the strategy to classify firms as financially constrained or not on the basis of firms' relative ranking in the distribution of variables arguably related with the availability or the cost of internal vs. external finance. Following a debate originated within the financing constraint to investment literature (Fazzari et al., 1988, Kaplan and Zingales, 1997, 2000), most studies focused on sensitivity of growth to cash flow as a measure of FC, but other variables such as leverage, availability of collateral, interest coverage have also been used. More recently, the attention has shifted toward measures of FC that, first, account at the same time for several dimensions of firm financial structure, and, second, are able to capture the different degree of FC, avoiding a simple binary categorization into constrained vs. non constrained firms (Cleary, 1999, Lamont et al., 2001, Whited and Wu, 2006, Musso and Schiavo, 2008). An alternative is to resort to survey data where firms/entrepreneurs are asked to give a self-assessment of the difficulty they face to access financing from banks or other institutions (Kaplan and Zingales, 1997, Winker, 1999, Angelini and Generale, 2008, Campello et al., 2009). None of the proposed measures is free from criticisms, though, and there is no clear consensus on how different measure can impact on the analysis. Relying upon standard business registry data inevitably gives an indirect measure of FC, assuming that poor records of firms with respect to the chosen variables get translated into banks' unwillingness to grant credit. Even if closer to answer the question whether a firm has been actually constrained or not when asking for credit, survey based measures of FC have their own limitations. First, they suffer from misreporting or sample selection bias whose effect is difficult to quantify. Second, by capturing the credit seekers' opinion on the actual binding of FC phenomena, they look at the demand side of credit relations, while one might argue that, given the strong asymmetries characterizing credit markets, the crucial role in determining FC is played by credit suppliers' opinion on the credit seeker.

A second major limitation characterizing the study of FC effects arises from the difficult identification of the many possible channels through which FC can affect size-growth dynamics of firms, over both the short and the long run. Short run effects concern how FC interplay with accumulation of growth opportunities. These have been at the core of the large, and more traditional, body of studies where some proxy of FC appears among the covariates in standard regressions of size on growth. The conclusion of these studies is typically that growth of younger and smaller firms is more severely affected by financing problems. Long run effects then pertain the question whether FC are able to significantly affect the size structure of business firms within an industry. Within regression analysis this is partially achieved by studying whether size evolves as an integrated process or not. A more general approach is instead to look at the evolution of the firm size distribution (FSD) over time or comparing the shape of the FSD for firms of different age. This topic is of more recent interest, and the scant evidence on the issue is contrasting. Cabral and Mata (2003) find that the evolution of FSD is determined by firms ceasing to be FC, while Fagiolo and Luzzi (2006) and Angelini and Generale (2008) conclude that FC are not the main determinant of the FSD evolution. Notice also that part of explanation for such seemingly contrasting evidence can come from the different proxies of FC employed. Cabral and Mata (2003) measure FC with age, assuming younger firms are constrained, while the other two studies proxy FC with measures based either on a traditional proxy of liquidity constraints (cash flow, in Fagiolo and Luzzi, 2006) or on survey data (Angelini and Generale, 2008).

The literature as it stands leaves many open questions on other channels which might be important in understanding if and to what extent FC affect size and growth of firms. Regression analysis can only say something on the effect of FC on the central moment of the firm growth rate distribution (FGRD), that is on average growth within an industry. It seems however natural to question whether FC have more general effects on growth rates. A well known result of empirical studies on firm dynamics states that the relationship between size and growth tends to be heteroskedastic, meaning that firms of different size display different variance of growth, with smaller firms experiencing more volatile growth rates. There is typically no attempt to understand if FC do have a role in producing this stylized fact, while heteroskedasticity is typically viewed as factor to wash away to obtain consistent estimates. But one can go a step further. Indeed (see Campello et al., 2009) report clear evidence that firms, when facing difficulties in raising external finance, undertake composite reactions: they tend to bypass attractive investment projects, display a much higher propensity to sell off productive assets as a way to generate funds, and the two effects seem exacerbating during economic crises. This points at a further question, so far largely neglected, concerning if and to what extent FC affect the the overall shape of the growth rates distribution.

In this paper we propose a novel approach to measure FC based on credit ratings, and develop a framework which is flexible enough to allow for a consistent analysis of the many possible FC channels, working over both the shorter and the longer run. Credit ratings, by their very definition, do not suffer from biases inherently affecting survey measures, at the same time sharing the advantages of multivariate indicators of FC based on hard data, especially in terms of the opportunity they offer to have a proxy for different degrees of FC. What is however peculiar of credit ratings is that they do not only provide a picture of a wide range of potential sources of financial problems. They also represent an opinion of credit suppliers on the ability of firms' to meet obligations, thus getting closer to measure whether credit is likely to be granted or not in practice. Some specific features of the credit rating index we are going to employ in this work render this consideration even more compelling, and makes us confident that our rating index actually serves as a benchmark for lending decisions of banks.

Exploiting credit ratings, we build classes of firms subject to different strength of FC and investigate to what extent short and long run dynamics of size-growth evolution vary by FC classes. On a general level, by combining distributional analysis of firm size and firm growth with regression analysis, our study provides a first attempt to reconcile the multiple effects of FC within a consistent framework. On a more specific ground, we provide novel evidence on the interplay of FC and age in determining the variability of growth shocks across firms of different size, and on the way different degrees of FC can affect the shape of both the firm size and firm growth rates distributions. Identification of such effects require suitable statistical techniques able to account for distributional asymmetries. In turn, distributional properties of the data inform the modeling and estimation strategy in regression analysis, where we also include controls for other dimensions which are likely to play a role in firms' financing.

Our findings suggest that FC do play a relevant role. Over the short run, they create a threshold effect either preventing attractive growth opportunities to be enjoyed by FC firms, or further deteriorating the growth prospects of slow growing firms. This effect gets translated into diverse patterns of growth across the different FC classes, and results, over the longer run, into diverse evolution of the firm size distribution.

The paper is organized as follows. Section 2 presents details on our measure of FC and reports descriptive evidence on the relevance of FC phenomena in our database. In Section 3 we develop the framework offering a solid guidance in targeting our empirical investigation and in interpreting our results. Section 4 investigates the effect of FC on the evolution of FSD, on the volatility of growth by different sizes, and on the shape of the FGRD. Section 5 presents the results of regression analysis and performs robustness checks. In Section 6 we summarize our findings and conclude.

2 Financing constraints: definition and basic facts

In this paper we employ a large database of Italian firms maintained by the Italian Company Account Data Service (Centrale dei Bilanci, CeBi). CeBi was founded as a joint agency of the Bank of Italy and the Italian Banking Association in the early 80's with the institutional task of providing assistance in supervision of risk exposure of Italian banking system. Nowadays CeBi is a private company owned by major Italian banks, which continue to exploit its services in gathering and sharing information on business firms. The long lasting institutional role of CeBi ensures high levels of data reliability, substantially limiting problems of measurement errors. Our dataset is of a business register type, collecting annual reports for *limited liability* firms which, under Italian Civil Law, face legal obligation to make their accounting publicly available. The dataset includes financial statements and balance sheets data of virtually all limited liability firms, being therefore highly representative of this type of firms. Concerning Manufacturing, which is the macro-sector we are going to focus on in the analysis, the dataset accounts for about 45% of total employment and about 65% of aggregate value added, as reported in the National Accounts data issued by Eurostat.[1] In the present study we have access to data over the period 2000-2003, for approximately $200,000$ firms.

For each firm, only a subset of the original list of variables included in the financial statement is made available to us. We compute Age from the year of foundation of each firm and we use real Total Sales as a proxy for size.[2] Our choice to prefer Total Sales over Number of Employees as a proxy for size is based on the consideration that, in the Italian accounting system, employment is reported only in the notes to financial statements and it is likely to be affected by less reliable annual updates. Notwithstanding this choice, kernel estimates of real sales densities by age class, reported in Figure 1, broadly confirm the basic stylized fact observed in previous studies proxying size with employment. The FSD is right skewed and both the mode and the width of the distribution increase with age. This visual impression is confirmed by running a Fligner and Policello test for stochastic dominance: the FSD of older firms dominates those of younger firms meaning that a firm randomly drawn from the group of older firms is, with a probability higher than 50%, bigger than a firm randomly extracted from the group of younger firms.[3] However, only relying on visual inspection makes it difficult to provide a precise statement on the validity of a second common piece

[1] Pistaferri et al. (2010) obtain similar figures, reporting that CeBi data represent about 50% of total employment, and around 7% in terms of number of firms.

[2] Nominal sales are deflated via 3-digit sectoral production price indexes made available by the Italian Statistical office, base year 2000. A basic cleaning procedure to remove few anomalous observations is applied. See Appendix 7 for details.

[3] This test is presented in Fligner and Policello (1981), while Bottazzi et al. (2008) provide details on the interpretation of the test in case of asymmetric samples.

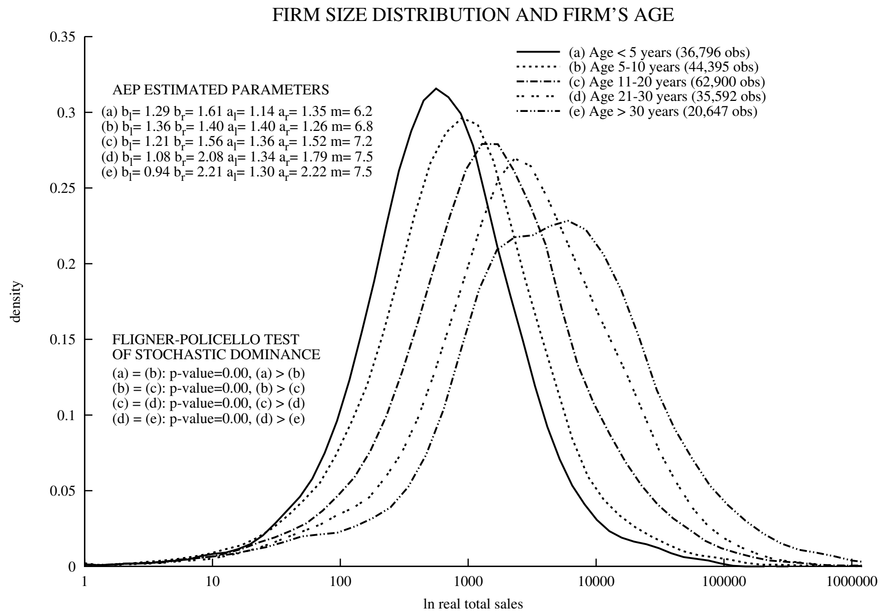

Figure 1: FSD and firm's Age. Densities estimates are obtained using the Epanenchnikov kernel with the bandwith set using the simple heuristic described in Silverman (1986). Pooled data from 2000,2001,2002 and 2003.

of evidence reported in this literature, namely that the degree of skewness of the FSD diminishes with age.[4] A preliminary remark is that when using Total Sales, instead of Number of Employees, as a proxy for size, the FSD appears to be more symmetric (cfr. again Angelini and Generale, 2008). In order to provide a meaningful quantitative assessment of this issue, we consider the Asymmetric Exponential Power (AEP) distribution. The AEP is a flexible 5-parameters distribution where (b_l, b_r) are left and right shape parameters, (a_l, a_r)left and right scale parameters while m is a location index. Notice that $b = 2$ represents a Gaussian tail while $b < 2$ identifies fat-tailed distribution. [5] The estimated AEP parameters reported in Figure 1 suggest that the evolution of FSD skewness reveal two different dynamics in the two tails two tails, which describe the size dynamics of small and big firms. The left tail becomes fatter as age increases (b_l decreases while a_l is approximately stable), simply capturing the well known result that firm size tends to increase with age. Further, we observe a shift of probability mass from the right tail to the central part of the distribution (both a_r and b_r increase with age), in agreement with another expected pattern that size cannot grow indefinitely. Altogether these evidences clearly clarify that age exerts an asymmetric effect on the evolution of the FSD, impacting differently in the two tails.[6]

The key variable for the purposes of the present paper is the rating index that CeBi produces for all the firms included in the dataset. Traditionally rating indexes are considered to yield a summary ranking of the revealed financial and economic performances of firms. In this paper we exploit ratings according to a different perspective, which considers them as an "opinion [of credit suppliers] on the obligor's *overall* capacity to meet its financial obligations"(Crouhy et al., 2001). In line with this interpretation we use CeBi credit ratings to proxy for financing constraints. Our proxy meets all

[4]See for instance Cabral and Mata (2003) and Angelini and Generale (2008).

[5]Appendix A provides details on the definition and estimation of the AEP.

[6]Notice that the alternative distribution used in Cabral and Mata (2003), the Extended Generalized Gamma distribution, possessing an unique shape parameter, does not allow to independently account for the behavior of the FSD in the left and right tails.

the requirements identified in the literature concerned with the construction of a meaningful measures of financial constraints (Cleary, 1999, Lamont et al., 2001). First, it results from a multivariate score analysis, thus summarizing a wide range of dimensions of firms' financial situation. Second, it is recomputed in each year, and thus allowed to change over time. Third, it does not force the researcher to work with a binary categorization into constrained vs. non-constrained firms; on the contrary it allows to capture different degrees of difficulty in accessing external funds. These features are shared, by construction, by other credit ratings, issued by well-known international agencies such as Moody's, Standard & Poor's or Fitch.

Our rating index possesses, however, some peculiarities which make it particularly attractive as a proxy of FC. A first advantage is that our rating is available for all the firms included in the dataset, while using credit files from international rating institutions would therefore bias the scope of analysis towards a smaller and much less representative sub-sample of firms. A second peculiar characteristic of our index rests in its status of 'official credit rating', due to the tight link established between CeBi and major Italian banks since its foundation. This has resulted into an heavy reliance of banks on CeBi ratings: it is virtually true that a firm with very poor rating is not likely to receive credit. Our measure of financial constraints, therefore, does not only yield a picture of the financial fragility of firms, but also represents an actual forecast of the ability to raise external finance.

We conclude the section by showing some descriptive evidence, featuring a first assessment of the FC phenomena in our database. We define three classes of firms subject to different degrees of financial constraints: Non Financially Constrained (NFC), Mildly Financially Constrained (MFC) and Highly Financially Constrained (HFC) firms.[7] The assignment to each class is made considering one-period lagged values of ratings. This is motivated by the simple observation that the rating index is updated at the end of each year: it is therefore the rating in t-1 that is relevant for credit suppliers when they have to decide about whether to provide credit or not in year t.[8]

Table 1 shows that, partially contrasting a result reported in Angelini and Generale (2008), financing problems represent a relevant phenomenon: 10% of the whole sample is affected by severe difficulties in raising external resources (cfr. the HFC class), while almost half of our sample (48%) faces less severe, but yet relevant problems (cfr. the MFC class). Second, FC represent a pervasive phenomenon, affecting firms of different sizes and ages: more than 5% of old firms are in the HFC class, and the mean size of HFC firms is comparable with the corresponding mean size in the other two classes.[9] However, confirming a robust finding of the literature, the effect of FC is stronger among young and small firms: 20% of young firms are HFC, against the 5% found in the group of older firms, and the median size of HFC firms is, in all age classes, almost one third smaller than the corresponding median size of firms which do not exhibit particular problems in collecting external resources.

[7]The CeBi index ranks firms with a score ranging from 1 to 9, in decreasing order of creditworthiness: 1- high reliability, 2-reliability, 3-ample solvency, 4-solvency, 5-vulnerability, 6-high vulnerability, 7-risk, 8-high risk, 9-extremely high risk. Our NFC, MFC and HFC class correspond, respectively, to firms rated from 1 to 4, from 5 to 7, and rated 8 or 9. Notice also that the ranking is an ordinal one: firms rated as 9 are not implied to have 9 times the probability of going default as compared to firms rated with a 1.

[8]To check the sensitivity of our results we also consider two alternative assignment procedures, described in details in Appendix 7. The main messages of the present paper are not influenced by the choice of the assignment procedure. Results are available upon request.

[9]The very high mean found within HFC old firms, 47.760, is explained by the presence of a single very big firm (actually the biggest in the dataset) which is old and HFC in the observed years. The mean size falls to 18,415 if we drop this firm from the sample.

Table 1: FINANCIAL CONSTRAINTS BY AGE CLASSES

	Whole Sample		Non Financially Constrained		Mildly Financially Constrained		Highly Financially constrained	
Firm's age (years)	Number of firms	Size: mean (median)	Number of firms (percentage of age class)	Size: mean (median)	Number of firms (percentage of age class)	Size: mean (median)	Number of firms (percentage of age class)	Size: mean (median)
0-4	38,020	1.795	10,356	1.804	20,408	1.970	7,256	1.293
		(0.606)	(27.2)	(0.525)	(53.7)	(0.719)	(19.1)	(0.449)
5-10	52,150	3.369	18,269	4.115	27,862	3.248	6,019	1.666
		(0.860)	(35.0)	(0.844)	(53.4)	(0.995)	(11.5)	(0.439)
11-20	62,977	7.093	29,130	8.210	29,408	6.400	4,439	4.354
		(1.522)	(55.9)	(1.606)	(46.7)	(1.663)	(7.0)	(0.525)
21-30	35,579	10.139	18,966	11.147	15,080	9.544	1,533	3.520
		(2.674)	(53.3)	(2.719)	(42.4)	(2.921)	(4.3)	(0.696)
31-∞	20,645	25.917	11,374	26.600	8,213	22.157	1,058	47.760
		(4.516)	(55.1)	(4.919)	(39.8)	(4.764)	(5.1)	(1.345)
Total	209,371	7.577	88,095	9.614	100,971	6.386	20,305	4.662
		(1.301)	(42.1)	(1.548)	(48.2)	(1.371)	(9.7)	(0.494)

Size as real sales, millions of euro.

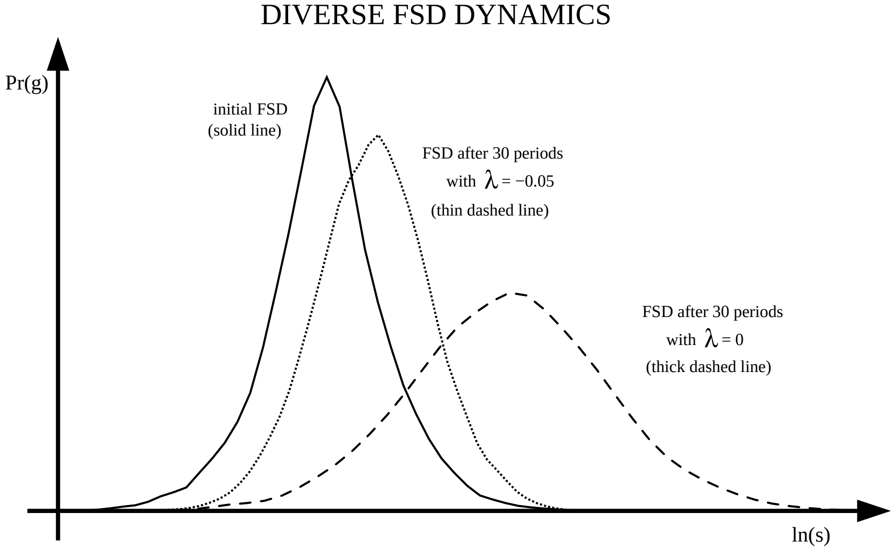

Figure 2: Evolution of the FSD for two different values of the autoregressive coefficient λ in equation (2).

3 A simple theoretical guidance

In this section we outline a framework that offers a simple theoretical guidance in targeting our empirical investigation and in interpreting our results on the effects of FC on firm dynamics. Building upon the classical work by Gibrat (1931) the traditional way of describing the size-growth dynamics of firms evolution of firm size has been to model the evolution of size through a simple process of the form

$$s_t \;=\; s_{t-1} + \epsilon_t \quad , \tag{1}$$

where s is the logarithm of firm size and ϵ are growth shocks.[10]

Notwithstanding its simplicity, this model has been shown to yield a good description of firm dynamics, at least in a first approximation. Indeed one does not observe any robust relationship between size and the average rate of growth (see among others Mansfield, 1962, Kumar, 1985, Hall, 1987, Bottazzi and Secchi, 2003). However, more recent empirical investigations has identified three main departures from the Gibrat's benchmark (see Lotti et al., 2003, for an in-depth review of the empirical literature). First, smaller (surviving) firms tend to grow faster. Second, age, while exerting a negative effect on growth, increases the surviving probability of firms. Third, the standard deviation of growth shocks falls as firm size increases. Such deviations from the simple Gibrat's model can be easily taken into account by modifying equation (1) as follows

$$s_t - s_{t-1} = c + \lambda\, s_{t-1} + \sigma(s_{t-1})\, \epsilon_t \quad , \tag{2}$$

where c is a drift term, λ allows for the presence of autoregressive effects, and $\sigma(s_{t-1})$ represents the standard deviation of growth shocks as dependent from size.

A quite natural and flexible approach to exploit this extended Gibrat's setup for the study of financing constraints effects is to allow for FC class specific patterns. This is easily obtained through a further extension of model (2) as

$$s_t - s_{t-1} = c_{FC} + \lambda_{FC}\, s_{t-1} + \sigma_{FC}(s_{t-1})\epsilon_{FC,t} \quad , \tag{3}$$

[10]Firm subscript has been removed for simplicity.

where financing constraints are now allowed to have an effect through four different channels shaping the size-growth interplay: the drift term c, the autoregressive term λ, the heteroskedastic term $\sigma(s_{t-1})$ and the growth shock ϵ.

This extension leaves us with a framework which is consistent with the empirical evidence available and allows to characterize the evolution of firm size on different time scales. On the one hand, the λ coefficient determines the long-run dynamics of s_t, defining the asymptotic shape of the FSD. On the other hand, equation (3) is also able to account for short-run features characterizing the evolution of size, which would be reflected in either the statistical properties of the distribution of the shocks ϵ or in the heteroschedasticity term $\sigma(s_t)$. In turn, the dependence of the variance of growth on size can be easily modeled to be consistent with the available empirical evidence.[11]

The coefficient λ captures the long-run dynamics of size, defining the asymptotic shape of the FSD. Too see how, if we neglect, for the sake of simplicity, the FC subscript and the heteroskedasticity correction, the variance of the size distribution V evolves in time from $t = 0$ to $t = T$ according to

$$V_{s_T} = (1 + \lambda)^T V_{s_0} + \frac{(1 + \lambda)^T - 1}{\lambda} V_\epsilon$$

where V_ϵ is the variance of the shocks. [12]

In the Gibrat's benchmark $\lambda = 0$ and, thus, the time evolution of s_t follows a unit root process converging to a Lognormal FSD with indefinitely increasing variance and zero mean.[13] Economic intuition suggests however that difficulties in accessing external financial resources should prevent firm size of constrained firms from developing according to the benchmark, basically impose constraints on the ability to enjoy growth opportunities. If this is the case, one expects $\lambda_{FC} < 0$ for more severely constrained firms. Accordingly, the size evolution of these firms should follow a mean reverting process, with small firms growing faster than big firms, and their FSD should converge in probability to a stationary distribution with finite variance $V_\epsilon/|\lambda|$. Figure 2 shows that even small differences in the value of λ, can produce significantly different FSD evolution in the long-run.

Next, if the interest lies in the short-term effects of financial constraints, then one has to study the distributional properties of growth shocks, and the way in which they are "loaded" in the evolution of size over time. These would result in FC class specific differences in the properties of c_{FC}, $\sigma_{FC}(s_{t-1})$ and ϵ_{FC}. Essentially, c and the function σ account for effects of FC that affect the positioning and variance of the firm growth rates distribution, but do not affect the shape of the entire distribution of growth. Differences in c across FC classes captures simple "location-shifts", with the growth rates distribution of more constrained firms presumably moving leftward as compared to other classes. On the other hand, FC class specific differences in the scaling term σ_{FC} would be revealing of a "scale-shift effect", suggesting that FC produces a change in the variance of the distribution. The standard approach, widespread in the literature, of estimating augmented Gibrat's type of equations like (2) with FC related variables among the regressors can only capture these two types of FC effects, implicitly assuming that FC do not affect on the overall shape of the FGRD.[14] The assumption is obviously quite restrictive, and there are no clear reasons why these should be case. Quite the contrary, there are many pieces of evidence that appear to be in sharp contrast with this presumption. Many studies, coming from various streams of research, have indeed testified that FC play a complex and structured role on firms' dynamics, affecting several dimensions of firms'

[11] See section 4.2 for a complete discussion of this issue.

[12] The same is true for all existing central moments. See Section 7.3 for a formal derivation

[13] This is due to the introduction of the drift term c.

[14] The alternative to enter FC classes as dummy variables capturing whether a firm is constrained or not (cfr. for instance Angelini and Generale, 2008), can only account for FC class dependent shifts in the constant term, thus suffering from a similar limitation.

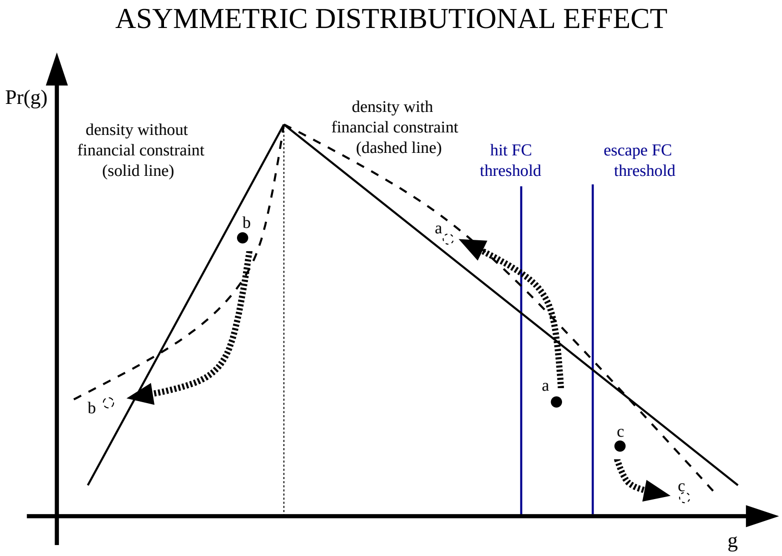

Figure 3: Possible effects of financing constraints on the growth rates distribution.

behavior and decisions, like investment/divestment strategies (Fazzari et al., 1988, Devereux and Schiantarelli, 1990, Bond et al., 2003), cash management policies (Campello et al., 2009), or R&D and innovation strategies (Hall, 2002, Brown et al., 2009). Pointing in the same direction are the results of a recent study based on a cross-country survey on CEOs' management of credit problems during the last crisis (see Campello et al., 2009). There is clear evidence that firms, when facing difficulties in raising external finance, undertake composite reactions: they tend to bypass attractive investment projects, display a much higher propensity to sell off productive assets as a way to generate funds, and the two effects seem exacerbating during economic crises. Going back to our framework, it would be quite simplistic to assume that all these different reactions to FC can be accounted for by location-scale shift effects. What seems more realistic to expect is that FC should bring about changes in the overall shape of the FGRD.

But then, what kind of predictions on the effects FC on the shape of FGRD can one draw from the qualitative evidence just outlined ? Figure 3 shows a graphical illustration of some apriori plausible patterns. A series of recent studies (cfr. Stanley et al., 1996, Bottazzi and Secchi, 2006a), which did not explicitly consider the effects of financial constraints, have shown that the distribution of the growth shocks follow a Laplace distribution. The solid line in Figure 3 takes this finding as a benchmark case.[15] The dashed line represents the shape that the distribution of the ϵ could assume under the influence of binding FC, according to three types of effects suggested by appreciative empirical evidence on composite reaction of firms to financing problems. The first one is a "pinion the wing" effect: FC prevent firms facing potentially good growth opportunities from actually catching some of them, at least beyond a certain threshold, forcing firms to bypass or postpone investment projects. These firms, even if still able to enjoy positive growth in presence of FC, would have had much higher growth records if not hit by FC (cfr. case labeled 'a' in Figure 3). A second possible pattern is caused by "when it rains, it pours" effect, predicting that firms who are already facing scarce growth opportunities without FC will experience further deterioration of their ability to catch good growth opportunities in presence of credit constraints problems, for example because they are forced to sell productive assets (cfr. case labeled 'b' in Figure 3). Third, and finally, one might not exclude a-priori the existence of firms who are able to escape difficulties in raising external finance

[15] On a log-scale this distribution presents the peculiar tent-shape displayed in Figure.

in some way, for instance by undertaking forms of ownership and/or management restructuring that helps them in better matching banks' credit requirements, or by acquiring financial resources outside the banking system (via venture capitalists or business angels). Plausibly, these firms must be those able to access exceptionally high growth opportunities (beyond the "escape FC threshold"), which they continue to enjoy even when they are hit by FC (cfr. case 'c' in Figure 3).

The remaining of the paper is aimed at deeply exploring the set of predictions on the effects of FC suggested by our theoretical framework. We will combine investigations of the distributional properties with regression analysis, shedding light on the overall effect of FC on size-growth dynamics of firms.

4 Distributional analysis

This section explores the effects of FC on distributional properties of size and growth. The properties of size distribution highlight important long-run effects of financial constraints. Next we focus on the possible effects in the short-run. We first investigate if FC affect scaling relations between variance of growth shocks and size firms size. Then, after properly taking into account this form of heteroskedasticity, we check if and to what extent FC are able to influence the shape of the firm growth rates distributions.

4.1 Firm size distribution, age and financial constraints

In Figure 4 we report kernel estimates of the FSD for the non, mildly and highly financially constrained firms directly comparing young (less than 5 years) and old (more than 30 years) firms in each class.[16] Visual inspection of the results (top left and right, and bottom left panels) points out a first, quite apparent piece of evidence: the evolution of the FSD is rather similar for NFC and MFC firms, while the FSD of HFC firms seems to display a quite peculiar dynamics. This peculiarity concerns essentially two aspects of the evolution of the FSD: the intensity of the location-scale shift effects and the progressive Gaussianization of its right tail.

As apparent from the plots, both location and scale shift effects are much milder among HFC firms than in the other two classes. This is confirmed in the bottom-right panel, where proxying the location and the scale of the FSD with the median size and the right width parameter of the AEP a_r, we explore how these two measures vary by age and FC classes. The values of both measures are very close across FC classes when firms are young. Then, as age increases, one identifies two diverging trends, one common to the NFC and MFC classes, and a second specific to HFC firms. The median sizes of NFC and MFC firms gets more than decupled passing from young to old firms, while the median size of HFC firms increases only by a factor of 5. Similarly, if we look at the estimated AEP right width parameter, a_r, we observe a large increase with age in the size dispersion of for NFC and HFC firms, while the increase in the dispersion for HFC, even if present, appears modest.

The same diverging trend also emerges if we take the estimates of b_r, the parameter capturing the right tail behavior. As age increases, from very similar values for young firms ($\sim$1.4, $\sim$1.7 and $\sim$1.6 for the NFC, MFC and HFC class respectively), the estimated coefficients polarize into two groups when old firms are considered. On the one hand, old NFC and old MFC firms display values of b_r close to 2 and hence approximately consistent with an Gaussian tail. This is not the case, on the other hand, if we look at old HFC firms, where the estimated b_r drops from 1.6 to 0.9.

[16]Other age classes are not reported for the sake of clarity.

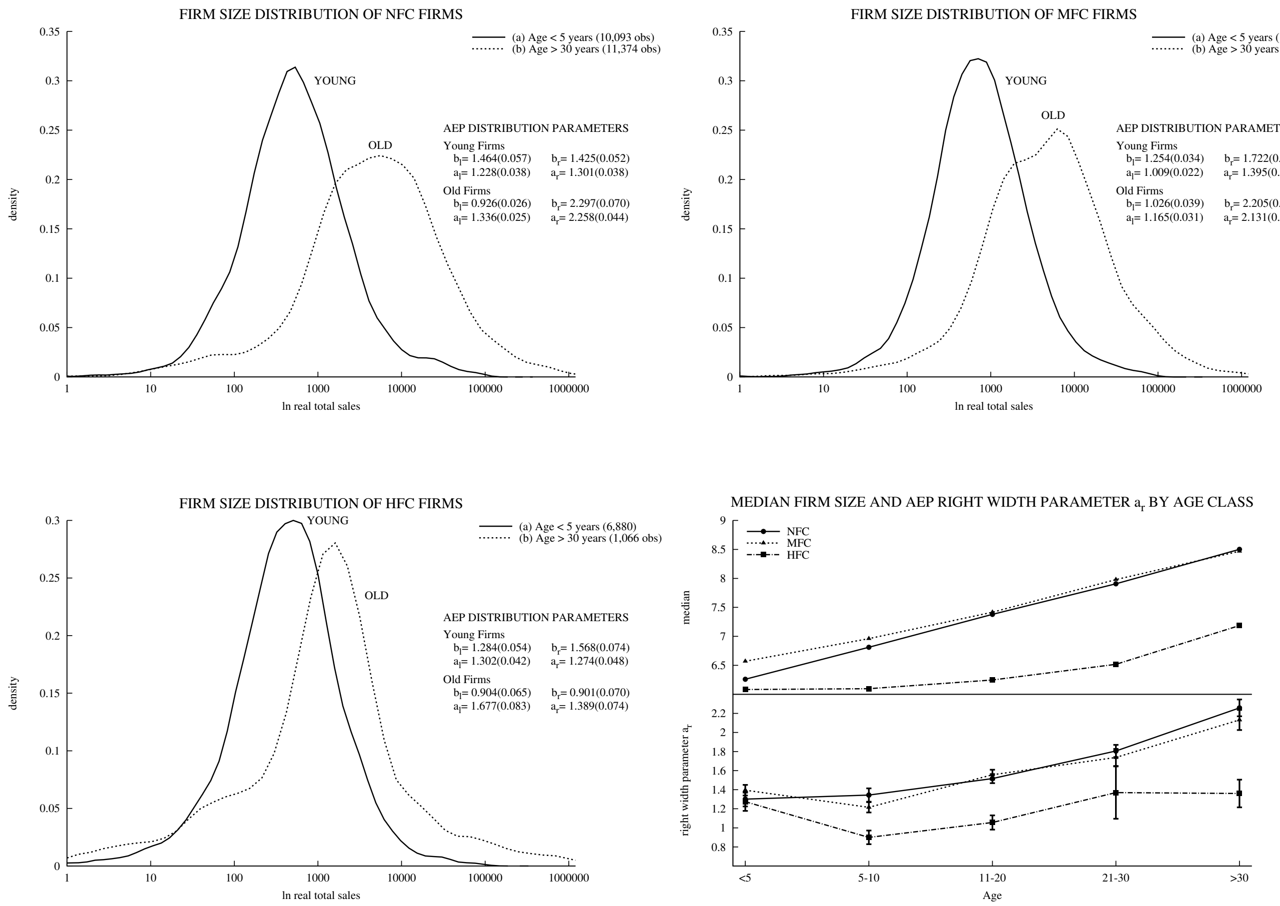

Figure 4: Kernel density of the FSD for young (less than 5 years) and old (more than 30 years) firms in different FC classes. Estimates are obtained using the Epanenchnikov kernel with the bandwidth set using the simple heuristic described in Silverman (1986). Pooled data over the period 2000-2003. Bottom right panel reports the median firm size and the AEP right width parameter, a_r, by age class. Details on the AEP are in Appendix 7

Overall, the very weak location-scale effects together with lacking Gaussianization in the right tail observed for the HFC are easily interpretable in light of the framework described in the previous section. Both effects are indeed compatible with the conjecture that FC phenomena produce a significant autoregressive component in size-growth dynamics, implying the same kind of FSD evolution reported in Figure 2 when $\lambda_{FC} < 0$.

4.2 Scaling of variance, age and financial constraints

Since Hymer and Pashigian (1962) it has been repeatedly shown on different databases that the variance of firm growth rates does decrease with size (among other see Amaral et al., 1997, Bottazzi and Secchi, 2005). In this section we explore if and to what extent this relation is affected by the strength of financial constraints. Beyond possessing a theoretical interest *per se*, the existence of this scaling effect in the data also represents a serious problem when one investigates the statistical properties of the FGRD. Indeed, if the standard deviation of growth process depends on the size of the firm, then pooling the growth rates across firms in different size classes amounts to consider a "mixture" of different densities, thus resulting in spurious properties of the FGRD (see for example Bottazzi

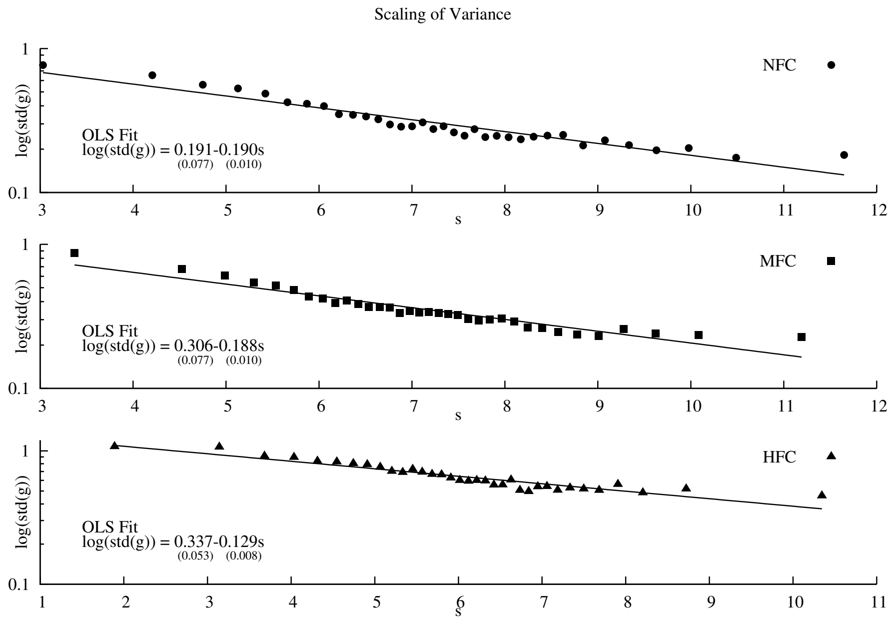

Figure 5: Binned (35 bins) empirical relations between the $\log$ standard deviation of growth and firm size for the three different FC classes. OLS estimates of equation (5) is also reported.

and Secchi, 2005). In this section we present a simple way to treat this form of heteroskedasticity.

Following previous studies, we start from the standard definition of growth in terms of log-differences of size

$$g_{i,t} = s_{i,t+1} - s_{i,t} \quad , \tag{4}$$

and we investigate the relation between the log standard deviation of g and firm size in different FC classes. In order to do that, for each FC class, we split firms in 35 size bins (quantiles) and we compute the log of within-bin standard deviation of growth rates.[17] Figure 5 confirms previous findings that the scaling of growth with size seems well approximated by a straight line. Hence, in agreement with this evidence, we model heteroskedasticity with an exponential law

$$\log(\sigma_{FC}(g_t|s_{t-1})) = \alpha_{FC} + \beta_{FC}\, s_{t-1} \tag{5}$$

where α_{FC} and β_{FC} must be estimated from the data. Maximum likelihood estimates of the parameters are reported on the same Figure 5, together with their standard errors.

The results, besides confirming that a linear approximation provides a good description of the variance-size relation (cfr. the solid line), reveal two important findings. First, the estimates of β, as expected negative in all the classes, display a negative relation with the degree of financial constraints. For NFC and MFC firms the estimates are very close to -0.19, a value which is strikingly similar to those reported in other studies on different data. This means that, in these two classes, the dispersion of growth rates of big firms (say with s_{t-1} equal to 10), is about three times smaller than the corresponding dispersion of small firms (say with $s_{t-1} = 4$). Among HFC firms, instead, β is about -0.13, implying a smaller reduction (of about two times) of growth dispersion between small and big firms in this class. In view of the conjectures put forward by our framework, such reduced differences in growth rates dispersion among small and big firms tells that FC seem to work by preventing firms from catching the highest growth opportunities. A second remarkable result is

[17]The whole procedure is very robust to the choice of the number of bins.

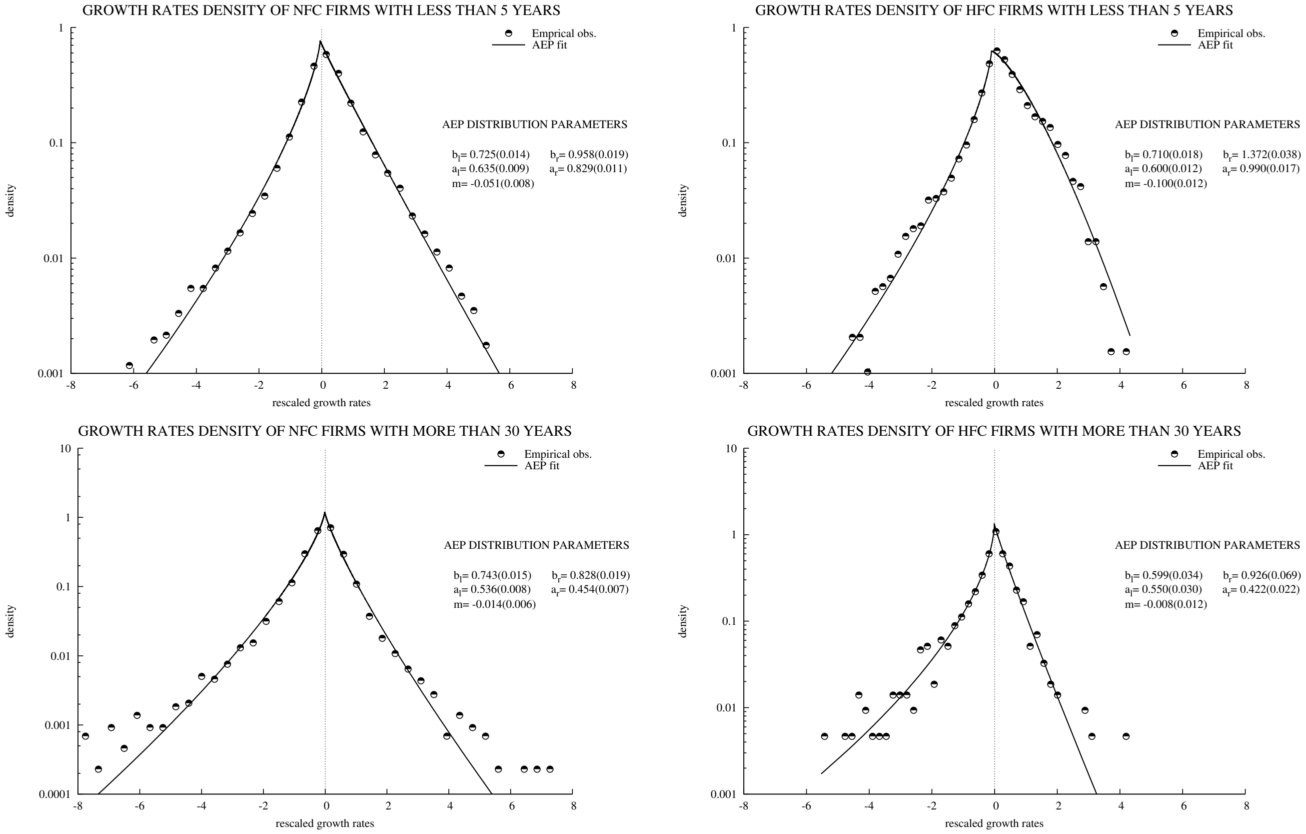

Figure 6: Growth rates distributions and financial constraints

that age is not playing any role in tuning the scaling relation of equation (5): α and β estimates do not substantially change if we estimate the relation by age class.[18]

4.3 Firm growth rate distribution, age and financial constraints (Under revision)

We then move on to the analysis of the effects of FC on growth shocks. Since the pioneering work of Stanley et al. (1996), it is well known that the firm growth rates distribution(FGRD), once heteroskedasticity has been properly modeled, possesses a peculiar tent-shape well approximated by a Laplace distribution.[19] Accordingly to the findings of the previous section, we define scale-invariant (or re-scaled) growth rates $\hat{g}$ as

$$\hat{g}_t = \frac{g_t}{e^{\alpha_{FC} + \beta_{FC} s_{t-1}}} \quad , \tag{6}$$

and we report in the four panels of Figure 6 the empirical distributions of rescaled growth rates $\hat{g}$ for young-NFC firms (top-left), young-HFC firms (top-right), old-NFC firms (bottom-left), and old-HFC firms (bottom-right).[20] In each panel we also show the AEP fit (solid line) and the corresponding estimates of the AEP coefficients b_l, b_r, a_l, a_r, capturing shape and width behavior in the left and right part of the distribution. Figure 6, besides confirming once again that a tent-shape

[18] Results not reported but available upon request.

[19] A first attempt to explain the emergence of this stylized fact, based on the idea of dynamic increasing returns, is in Bottazzi and Secchi (2006a)

[20] Distribution for MFC are not reported since they do not change our reasoning. They are available upon request.

represent a very robust property of the FGRD, show that FC have an apparent effect on the shape of the growth rates distribution. Before describing more in details this effect it worths noticing that even the simple identification of such an effect is a remarkable result *per se*: if it is reasonable to expect an apparent impact of FCs on firm size and on its distribution the same seems not true for the growth shocks. Indeed size is a stock variable that results from the integration of many growth shocks, hence cumulating the effects of FC over, possibly, long time horizon. On the contrary, it should be more difficult to identify the effects of FC on growth rates since they do not retain any memory of past effects of financing constraints.

Let us focus, first, on young firms (cfr. the two top panels in Figure 6). If we draw a comparison moving from the NFC to the HFC growth rates distribution, what we observe is a clear slim down of the right tail: there's a leftward shift of probability mass from the right tail to the central part of the distribution (b_r increases from about $\sim$0.96 for the HFC class, to almost 1.37 for the NFC). Correspondingly, also the right width increases: a_r goes from about 0.83 to about 0.99. On the contrary the left tails does not show any clear-cut difference among the two FC classes: the estimates of both a_l and b_l are very similar in the two FC classes when only young firms are considered. This evidence points at the existence of firms, typically young, that due to binding FC seem not able to fully exploit their growth opportunities thus reducing the fatness of the right tail (cfr. case "a" in Figure 3). When, instead, we focus on old firms (bottom panels of Figure 6) the picture changes. In this case the differences in the FGRD between financially constrained and non-financially constrained firms are stronger in the left side of their supports. The left tail is indeed fatter for the HFC class suggesting that FC bring about a shift of probability mass toward the left tail: b_l decreases from 0.74 to almost 0.60.[21] This points at the existence of firms (typically old firms) whose growth performances are reduced by binding FC that prevent them from pursuing their growth enhancing investment projects or even worse force these firms to sell their productive capacity to obtain liquidity (cfr. case "a" in Figure 3).

5 Regression analysis

We now turn to regression analysis. This allows us to provide further evidence on the effects of FC emerged so far, and enables to check whether some of the main conclusions reached survive the inclusion of potentially relevant determinants of size-growth dynamics overlooked in the graphical analysis.

We start with our baseline extension of standard Gibrat's framework, suggesting to model the time evolution of firm size as an autoregressive and heteroskedastic process of the form

$$s_t - s_{t-1} = c_{_{FC}} + \lambda_{_{FC}}\ s_{t-1} + \sigma_{_{FC}}(s_{t-1})\epsilon_{t_{FC}} \qquad . \tag{7}$$

where subscripts FC, as before, underlines that parameters can vary by FC class, thus allowing for FC class dependent effects.

The estimation strategy is guided by the statistical properties of the data emerged in the previous analysis. First, given the non-linear relation found between size and variance of growth rates, standard corrections of the estimates for heteroskedasticity in the residuals are doomed to fail. We therefore explicitly model heteroskedasticity via an exponential scaling equation, $\sigma_{_{FC}} = \exp(a + b\ s_{_{FC}})$. Second, to account for tent-shape and asymmetries of growth shocks revealed by the analysis of rescaled growth rates, we perform maximum likelihood estimates allowing the residuals to follow an Asymmetric Laplace Distribution.

[21] There is also an effect on the right part of the supports, qualitatively similar to that observed among young firms and resulting into fatter right tail of the NFC. For old firms, however, the effect is milder and almost negligible.

Table 2: AUGMENTED GIBRAT'S REGRESSIONS[a]

	FC CLASS	Model 1	Model 2A	Model 2B	Model 3
	NFC				
constant		0.010*(0.003)	0.075*(0.003)	0.060*(0.006)	0.053*(0.004)
$\ln(S_{i,t-1})$		-0.0002(0.0003)	-0.007*(0.001)	-0.008*(0.001)	-0.006*(0.001)
$\ln(Age_{i,t})$			-0.025*(0.001)	-0.025*(0.001)	-0.019*(0.001)
$\ln(Assets^{b}_{i,t-1})$			0.011*(0.000)	0.012*(0.001)	0.018*(0.001)
$\ln(GOM^{b}_{i,t-1})$			0.000(0.000)	0.001(0.001)	-0.003*(0.001)
$\ln(S_{i,t-2})$					-0.001(0.001)
$\ln(Assets^{b}_{i,t-2})$					-0.010*(0.001)
$\ln(GOM^{b}_{i,t-2})$					0.005*(0.001)
Time dummies		No	No	Yes	No
Sectoral dummies		No	No	Yes	No
Number of observations		85,382	85,382	85,382	51,181
	MFC				
constant		0.065*(0.003)	0.148*(0.003)	0.055*(0.007)	0.097*(0.004)
$\ln(S_{i,t-1})$		-0.007(0.0004)	-0.017*(0.001)	-0.008*(0.001)	-0.024*(0.001)
$\ln(Age_{i,t})$			-0.041*(0.001)	-0.041*(0.001)	-0.024*(0.001)
$\ln(Assets^{b}_{i,t-1})$			0.015*(0.000)	0.015*(0.001)	0.028*(0.002)
$\ln(GOM^{b}_{i,t-1})$			0.005*(0.000)	0.003*(0.000)	0.005*(0.001)
$\ln(S_{i,t-2})$					0.012*(0.001)
$\ln(Assets^{b}_{i,t-2})$					-0.018*(0.002)
$\ln(GOM^{b}_{i,t-2})$					-0.002(0.001)
Time dummies		No	No	Yes	No
Sectoral dummies		No	No	Yes	No
Number of observations		97,437	97,437	97,437	55,134
	HFC				
constant		0.175*(0.012)	0.396*(0.012)	0.259*(0.020)	0.158*(0.016)
$\ln(S_{i,t-1})$		-0.022*(0.002)	-0.046*(0.002)	-0.035*(0.003)	-0.047*(0.004)
$\ln(Age_{i,t})$			-0.109*(0.003)	-0.107*(0.003)	-0.022*(0.004)
$\ln(Assets^{b}_{i,t-1})$			0.037*(0.002)	0.039*(0.002)	0.048*(0.005)
$\ln(GOM^{b}_{i,t-1})$			0.006*(0.001)	0.005*(0.001)	0.010*(0.002)
$\ln(S_{i,t-2})$					0.016*(0.003)
$\ln(Assets^{b}_{i,t-2})$					-0.022*(0.004)
$\ln(GOM^{b}_{i,t-2})$					-0.004*(0.001)
Time dummies		No	No	Yes	No
Sectoral dummies		No	No	Yes	No
Number of observations		18,834	18,834	18,834	9,786

[a] Asymmetric Least Absolute Deviation estimates.
[b] Assets is proxied with Net Tangible Assets. Gross Operating Margin(GOM) has been transformed to avoid negative numbers.
[*] Significantly different from zero at 1% level.

Results by FC class are shown in Table 2 (cfr. Model 1).[22] The estimates point in the same direction of the evidences emerged from the analysis of FSD, providing further support that strong FC produce deviations from the Gibrat's benchmark. The estimates of λ are indeed non significantly different from zero for NFC and MFC firms, suggesting that an integrated process can represent a good approximation for the evolution of size in these two classes. Conversely, we find that λ is negative for HFC firms ($\sim$-0.02), revealing that strong FC give raise to a significant autoregressive component.

An obvious limitation of the simple framework adopted so far is that there might be important factors shaping size-growth dynamics which are left out from the analysis. The relatively short time dimension of the data does not allow to perform reliable panel estimates which would help to control for unobserved firm specific heterogeneity. However, we investigate several extensions of the model where we expand the set of regressors to control for potentially many relevant factors which we do observe. First, inclusion of firm age is mandatory, given the high correlation of age with size and the significant effects that age has shown to have on the distributional properties of both size and growth. Second, one wants to control for two dimensions which are likely to affect investment strategies of firms, interacting with FC in determining the financial resources available to the firms: availability of internally generated resources and availability of collateral goods. Concerning internal resources, the literature has traditionally focused on measures of cash flow. Since this is not available in the limited set of variable we can access, we take the logarithm of Gross Operating Margins (GOM), yielding a measure of profit margins generated by the mere operational activity of the firm. [23] Given the relatively high frequency of negative GOM in the sample (about 30%, in line with other studies on Italy, (see Bottazzi et al., 2008)), negative values of GOM have been transformed into a zero before taking logs: for the purpose of the analysis, indeed, negative or null availability of internal resources captures an equal need for a firm to completely rely upon external resources in financing its operation. The rationale behind inclusion of a proxy for collateral is that, as predicted by theory and confirmed by evidence (Angelini and Generale, 2008), the availability of hard capital can ease access to external finance. Accordingly, our measure of collateral is the stock of Net Tangible Assets (ASSETS), in logs. Taking these variables into account, we perform ALAD estimates of the following regression

$$\begin{aligned} s_t - s_{t-1} \;\; = \;\; & c_{FC} + \lambda_{FC}\; s_{t-1} + \beta_{1_{FC}}\; \ln(age_t) + \beta_{2_{FC}}\; \ln(GOM_{t-1}) + \\ & \beta_{3_{FC}}\; \ln(ASSETS_{t-1}) + \sigma_{FC}(s_{t-1})\epsilon_{t_{FC}}\;, \end{aligned} \tag{8}$$

where both GOM and ASSETS enter with 1-period lag, at least partially accounting for simultaneity of these variables, and we again model model heteroskedasticity via an exponential correction.[24]

The main effect of the inclusion of controls (cfr. Model 2A in Table 2) seems to be that deviations from the Gibrat's benchmark of $\lambda = 0$ are now observed in all the classes, according to a well known result repeatedly found in studies exploring augmented Gibrat's regression. Notwithstanding, and more interestingly for our purposes, the estimates of λ across the FC classes reproduce the pattern obtained before: deviations from the Gibrat's benchmark are much more sizeable within the HFC class, confirming that FC have a relevant effect in shaping size-growth dynamics of strongly

[22]Reported estimates refer to our usual assignment of firms to FC classes based on 1-period lag ratings. We also checked different assignment procedures, taking either the worst rating over the period of observation, or restricting to firms that never change rating over the sample period. Changing assignment rule does not affect the main conclusions, though.

[23]To be precise, considering the specificities of the Italian accounting definitions, the GOM is identical to the EBIDTA, measuring earnings net of financial flows, depreciation, amortization and taxes.

[24]As done for size, both GOM and ASSETS have been deflated with proper sectoral price indexes, at 3-digit level of industry disaggregation.

financially constrained firms. Interesting variation is also characterizing the effects of control covariates. Age displays a negative and significant coefficient in all classes, confirming the expectation that older firms grow generally less than younger firms. The magnitude increases with the strength of FC, revealing that such detrimental effect of age is even stronger for HFC firms. Also notice that age is the regressor with the stronger effect (bigger coefficient in absolute value). Next, ASSETS have a positive and significant effect, stronger for HFC firms: as it is reasonable, availability of collateral becomes more beneficial for growth when FC are stronger. Similarly, the availability of internal resources has some beneficial effect on growth only when FC are more severe, while it does not seem to be crucial for NFC firms (cfr. GOM is not significant for NFC, positive and significant for MFC and HFC). Notice however that, even when significant, the magnitudes are negligible in practical terms, suggesting that internal resources have (if any) a second order effect as compared to other regressors. The following column of Table 2 (cfr. Model 2B) shows that all of these patterns remain unchanged if we also include year and industry (2-Digit) dummies, controlling for time and sector specific effects.

Finally, we present estimates of a further specification (cfr. Model 3) where we also include 2-period lags of size, ASSETS and GOM. This allows for a further check of varying effects over time, and provide a further control for possible endogeneity of covariates at $t-1$.[25] The previous discussion concerning deviations from Gibrat's benchmark remain valid: the estimates of λ retain their signs and magnitudes, again displaying negligible values for NFC firms and then increasingly negative as FC become stronger. As largely predictable, second lag coefficients of GOM and ASSETS absorb part of the first lag effects, whose coefficients become generally smaller. The most significant reduction is observed in the age coefficient, whose magnitudes becomes now comparable with the effects exerted by other regressors, and also comparable across FC classes.

6 Conclusion

Credit ratings represent a particularly suitable measure to obtain a reliable proxy for financial constraints phenomena. They offer a way out to overcome the rather strict binary distinction into constrained vs. non-constrained firms traditionally employed in the literature, yielding a proxy of different degrees of credit problems. At the same time, ratings are heavily relied upon by banks and investors in granting and pricing credit lines, thus representing a benchmark or a key ingredients in banks' lending decisions. Taking advantage of these features of credit ratings, this papers provides new empirical evidence on the effects of financial constraints in determining, first, the evolution of firm size distribution, and second, the dynamics of firm growth. Our main contribution consists in providing a general and simple framework which enables to offer a consistent analysis of the diverse effects that FC exert on firms dynamics.

Concerning the evolution of the firm size distribution, extending some recent result in the literature, we find that firms experiencing more severe financial constraints display peculiar FSD evolution significantly different from that characterizing firms not suffering credit problems. This is confirmed when we allow for FC specific effects within a standard autoregressive model of logarithmic size: severely constrained firms display negative and sizeable autoregressive coefficient, even when controlling for age and other factors affecting financing decisions of firms. Second, concerning scale effects of growth with size, we find that variability of growth decreases with size in all FC classes, but the effect is milder for the more severely constrained firms. Third, once such scale effect is accounted for in measuring growth rates, our analysis reveal asymmetric effects of FC on the tails of

[25] Time and sector dummies are not considered in this case, given the blank effect of their inclusion on the other coefficients.

the growth rates distribution. In agreement with previous qualitative evidence, a “pinion the wing” effect, reducing the fatness of the right tails of the FGRD, coexists with a “when it pours, it rains” effect of FC, producing a shift of mass toward the left-tail. All of these findings build upon parametric methods allowing to precisely quantify different tail patterns, providing a further improvement over previous research.

On the interpretative side, our findings seem to tell a consistent story, revealing a relevant role of FC in shaping firm size and growth dynamics. Over the short run, our results lend support to the conjecture that FC create a threshold effect preventing constrained firms to fully catch the growth opportunities they face. On the one hand, there are firms (tipically young) that, despite facing attractive and potentially positive growth opportunities, are forced to grow less by FC. On the other hand, there exist firms (typically old) already experiencing slow growth that, due to FC, undertake a further deterioration of growth prospects, either because binding FC prevent them from pursuing some growth enhancing investment projects or because force them to sell their productive capacity to obtain liquidity.

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7 APPENDIX A

7.1 Cleaning anomalous observations

We remove from our sample few anomalous data. The cleaning has been performed using Total Sales as a reference variable: for each firm a missing values has been inserted, in place of the original Total Sales value, when the latter lied outside the interval

$$[Median(TS_t)/10; Median(TS_t) * 10] \quad t = 1998 \ldots 2003 \quad . \tag{9}$$

Descriptive statistics of Total Sales in different years before and after the cleaning procedure are reported in Table 3. It is apparent that our cleaning strategy does not introduce any qualitative change in our data.

Table 3: TOTAL SALES[a] DESCRIPTIVE STATISTICS

BEFORE CLEANING FILTER								
Year	Mean	Median	Std. Dev.	Skewness	Kurtosis	Min	Max	Obs.
2000	5700.82	1014.00	48730.09	57.89	4894.16	1.00	5634948.00	109689.00
2001	5972.90	1011.00	73679.67	141.82	29897.12	1.00	17547260.00	113405.00
2002	5804.92	973.00	67304.35	146.66	32359.62	1.00	16484840.00	116084.00
2003	5639.77	953.00	64724.22	147.42	32317.38	1.00	15803760.00	115777.00
AFTER CLEANING FILTER								
Year	Mean	Median	Std. Dev.	Skewness	Kurtosis	Min	Max	Obs.
2000	5754.55	1046.00	47700.57	58.99	5192.76	1.00	5634948.00	107250.00
2001	5878.64	1025.00	69435.93	159.48	37224.24	1.00	17547260.00	112036.00
2002	5806.96	992.00	67093.95	150.02	33371.72	1.00	16484840.00	113849.00
2003	5688.46	981.00	65417.79	147.67	32063.94	1.00	15803760.00	111810.00

[a] Nominal Total Sales in thousands of Euro.

7.2 Financial constraints classes: assignment procedures

We use the rating index present in our database to build classes of firms subject to different degrees of financial constraints. We distinguish 3 different classes: Non Financially Constrained (NFC), Mildly Financially Constrained (MFC) and Highly Financially Constrained (HFC). The general assignment rule, labeled "Lag 1", states that the NFC class at time t includes solvable firms with assigned rating at $t-1$ from 1 to 4, the MFC firms facing moderate financial fragility with rating at time $t-1$ in the range 5-7 while the HFC one contains firms with ratingat time $t-1$ equal to 8 or 9, namely firms characterized by an extremely low creditworthness.

In order to check the sensitivity of our results to the assignment procedure we consider 2 alternative procedures. In the first one, labeled "The Worst", we consider the whole time window of our database and we assign firm to classes on the base of the lowest rating they display. In the second one, labeled "Persistent", we assign firms to different classes only if they do not change their financial status over the whole time window (i.e. based on their ratings in different years they would always be included in the same FC class).

7.3 Asymptotic behavior of the autoregressive process

Assume the dynamics of firm size as described by (2), where the shocks ϵ are independent and identically distributed according to a probability density f with zero mean. Let s_0 be the initial size of the firm. Dropping for simplicity the heteroskedastic term, $\sigma(s_t) = 1$, and by recursive application of (2), the size s_T after T time steps can be written as the weighted sum of T independent random variables

$$s_T = (1+\lambda)^T \, s_0 + \sum_{\tau=0}^{T-1} (1+\lambda)^\tau \, \epsilon_{t-\tau} \ .$$

Consider the cumulant generating function of the size at time T, $\tilde{g}_{s_T}$, defined as the logarithm of the Fourier transform of the unconditional distribution

$$\tilde{g}_{s_T}(k) = \log \mathrm{E}[e^{ikS_t}] \ .$$

Due to the i.i.d. nature of the shocks it is immediate to see

$$\tilde{g}_{s_T}(k) = \tilde{g}_{s_0}((1+\lambda)^T k) + \sum_{\tau=0}^{T-1} \tilde{f}((1+\lambda)^\tau k)$$

where $\tilde{g}_{s_0}$ and $\tilde{f}$ are the cumulant of the initial size distribution and of the shocks distribution, respectively. Then if the initial side distribution and the shocks distribution possess the cumulant of order n, C^n, also the size distribution at time T possesses it, and one has, with obvious notation,

$$C^n_{s_T} = \left. \frac{\tilde{g}_{s_T}(k)}{dk} \right|_{k=0} = (1+\lambda)^T \, C^n_{s_0} + \frac{(1+\lambda)^T - 1}{\lambda} \, C^n_\epsilon \ .$$

The equation in Section 3 follows directly by noting that the variance is the second cumulant $V = C^2$.

7.4 Asymmetric Power Exponential(AEP) Distribution

The Asymmetric Exponential Power (AEP) is a new 5-parameter family of distributions introduced in Bottazzi and Secchi (2006b), able to cope with asymmetries and leptokurtosis and at the same time allowing for a continuous variation from non-normality to normality. The Asymmetric Exponential Power distributions is characterized by two positive shape parameters b_r and b_l, describing the tail behavior in the upper and lower tail, respectively; two positive scale parameters a_r and a_l, associated with the distribution width above and below the modal value and one location parameter m, representing the mode. The AEP density presents the following functional form

$$f_{\mathrm{AEP}}(x;\mathbf{p}) = \frac{1}{C} \ e^{-\left(\frac{1}{b_l} \left| \frac{x-m}{a_l} \right|^{b_l} \theta(m-x) + \frac{1}{b_r} \left| \frac{x-m}{a_r} \right|^{b_r} \theta(x-m) \right)} \tag{10}$$

where $\mathbf{p} = (b_l, b_r, a_l, a_r, m)$, $\theta(x)$ is the Heaviside theta function and where the normalization constant reads $C = a_l A_0(b_l) + a_r A_0(b_r)$ with

$$A_k(x) = x^{\frac{k+1}{x}-1} \, \Gamma\left(\frac{k+1}{x}\right) \ . \tag{11}$$

Bottazzi and Secchi (2006b) prove that the Maximum Likelihood (ML) estimates of the AEP parameters are consistent on the whole parameter space, and when sufficiently large values of the shape parameters are considered, they are also asymptotically efficient and normal. Moreover it is shown that with a sample size of at least 100 observations, the bias associated with ML estimates, although present, becomes negligible.